

The Conformal Transformation in General Single Field Inflation with Non-Minimal Coupling

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ABSTRACT: The method of a conformal transformation is applied to a general class of single field inflation models with non-minimal coupling to gravity and non-standard kinetic terms, in order to reduce the cosmological perturbative calculation to the conventional minimal coupling case to all orders in perturbation theory. Our analysis is made simple by the fact that all perturbation variables in the comoving gauge are conformally invariant to all orders. The structure of the vacuum, on which cosmological correlation functions are evaluated, is also discussed. We show how quantization in the Jordan frame for non-minimally coupled inflation models can be equivalently implemented in the Einstein frame. It is thereafter argued that the general N -point cosmological correlation functions (of the curvature perturbation) are independent of the conformal frame.

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1. Introduction

It has been by now well-established that inflation can naturally solve cosmological problems such as the flatness, homogeneity and monopole problems, thereby avoiding a fine-tuning for the initial condition of the history of our Universe. Inflation is also known to predict almost scale invariant Gaussian density perturbations. At the next stage of observation, the detection of non-Gaussianity [1]-[4] is one of the central concerns, since a detailed study of the non-Gaussianity and the shape of the bispectrum [5][6] in particular could constrain various inflationary models. It has been argued that in slow roll models of a single scalar inflaton the non-Gaussianity is negligibly small [7][8]. In the last several years there has been a lot of work exploring inflation models in which large non-Gaussianity could be produced. A partial list of such models includes DBI type inflation, k-inflation, curvaton scenarios, ghost inflation and so on.

In the present paper we will consider a general class of inflationary models in which a single inflaton field couples non-minimally to background geometry. In such a broad class of inflationary models, we can explore the classification of the shape of the bispectrum. It has been known rather well that the method of conformal transformation makes it possible to put the calculation of cosmological correlations into the minimal coupling case, and we will actually make full use of the method throughout this work. What we would like to add to this common wisdom is that the reduction to the minimal coupling case is in fact possible to all orders in perturbation theory. This paves the way for avoiding technical difficulties

associated with the non-minimal coupling to a considerable extent and for alleviating the need for separate studies.

The method of conformal transformation has been applied before in cosmology to discuss a higher derivative gravity theory [9]. It has been proved in [10] that the comoving curvature perturbation on slices orthogonal to comoving worldlines in the Jordan and Einstein frames coincides at the first order. It has also been shown in [11] that the curvature perturbation on uniform-density hypersurfaces agrees in these two frames up to second order. A couple of years ago Chiba and Yamaguchi [12] showed a very remarkable fact that all perturbation variables in the comoving gauge are conformally invariant to all orders of perturbation. More recently Gong et al. [13] have given another proof of the conformal invariance and have discussed its implications in the δN formalism. Motivated by the observation of Chiba and Yamaguchi and that of Gong et al. we now go one step further to show the usefulness of the conformal invariance of the perturbation and to present a coherent method to perform practical calculations of cosmological correlations in a general conformal frame.

The usefulness of the conformal transformation has been demonstrated in the context of cosmological perturbations by many people [14] - [18]. While we share basic ideas with these previous authors, we would like to emphasize that our argument can be extended rather easily to all orders in the cosmological perturbation in contrast with previous works. An all order argument is made possible by the fact that, due to our choice of gauge, the fluctuation part is separated from the conformal transformation factor.

In connection with our work, let us recall that Seery and Lidsey [19] and Chen et al. [20] investigated the shape of non-Gaussianity in a general Lagrangian of single field inflation models minimally coupled to gravity. In the case of arbitrary sound speed the shape is classified in terms of five parameters. Their work has been recently extended to the non-minimal coupling case by Qiu and Yang [21]. The computation of the cosmological correlation becomes necessarily difficult in the Jordan frame. This is particularly so when one has to solve equations of motion for the mode functions of the curvature perturbation. Our technique shows that the difficulties in the non-minimal case are only ostensible and that all of the calculations in [21] can be reproduced by a conformal transformation from those in [19] - [20] .

The present paper is organized as follows. After presenting the general single field inflation model in Sec. 2, we apply the conformal transformation and discuss connections between various quantities in the Jordan and Einstein frames in Sec. 3. In Sec. 4 we express the quadratic and cubic effective actions with respect to the curvature perturbation in the Einstein frame, and then we transform the action back into the Jordan frame in Sec. 5. The connection between the calculations in [19] - [20] and those in [21] is clarified there. In Sec. 6 the connection between the mode-functions in Jordan and Einstein frames is discussed. It is argued that the vacuum, on which correlation functions are computed, is identical in the two frames. The fact that N -point cosmological correlation functions agree in Jordan and Einstein frames is proved there. Our work is summarized in Sec. 7.

2. General single field inflation

In the following sections we discuss the general models that include a scalar field coupled non-minimally to gravity and go on to perform a conformal transformation to put the equations into a form which is the same as in the minimal coupling case. First we consider the following generalized Einstein-scalar gravitational system

$$S = \frac{1}{2} \int dt d^3\vec{x} \sqrt{-g} \left[\frac{1}{8\pi G} f(\varphi) R + 2P(\varphi, X) \right], \quad (2.1)$$

where we have introduced the notation

$$X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi. \quad (2.2)$$

Hereafter we omit the gravitational constant G by setting $8\pi G = 1$.

The functional $f(\varphi)$ of a scalar field φ coupled to the scalar curvature R can be arbitrary. The minimal coupling case corresponds to choosing $f(\varphi) = 1$; however attention has been paid in literature [11], [22]- [28] to non-minimal cases such as $f(\varphi) = 1 - \xi \varphi^2$, where ξ is a parameter. We wish, however, to proceed most generally without specifying a particular example of $f(\varphi)$. In (2.1) we have also introduced an arbitrary functional $P(\varphi, X)$, which can be expanded in power series in X as

$$P(\varphi, X) = -V(\varphi) + K(\varphi)X + L(\varphi)X^2 + \dots \quad (2.3)$$

The case of k-inflation [29] corresponds to $V(\varphi) = 0$. The non-canonical kinetic term of φ , which could be produced in underlying fundamental theories such as supergravity and superstring theories, has many physical implications. We will, however, proceed without explicit use of the expansion as in (2.3). In principle we could include higher derivative terms of φ as well. Effects of such higher derivative terms would be presumably suppressed by the ultraviolet cut-off scale. In so far as we consider the first derivative terms of φ , the action (2.1) is the most general.

The equations of motion for the scalar field φ and gravity turn out to be

$$\frac{1}{2} \frac{\partial f(\varphi)}{\partial \varphi} R + P_{,\varphi} + (P_{,XX} \nabla^\mu X + P_{,X\varphi} \nabla^\mu \varphi) \nabla_\mu \varphi + P_{,X} \nabla_\mu \partial^\mu \varphi = 0, \quad (2.4)$$

$$\nabla_\lambda \partial^\lambda f g_{\mu\nu} - \nabla_\mu \nabla_\nu f + f R_{\mu\nu} - \frac{1}{2} f R g_{\mu\nu} - P_{,X} \nabla_\mu \varphi \nabla_\nu \varphi - P g_{\mu\nu} = 0. \quad (2.5)$$

The derivatives of P with respect to X and φ are denoted by $P_{,X}$ and $P_{,\varphi}$, respectively. The classical solution $\varphi(t)$, which does not depend on spatial coordinates, is obtained by solving (2.4), and hereafter φ shall always denote the classical solution after choosing an appropriate gauge (as will be discussed in Eq. (4.4)).

3. Conformal transformation to the Einstein frame

Given the general action (2.1), it is useful to employ a conformal transformation to go to the Einstein frame, which will be simpler and more familiar for us to develop cosmological

perturbations. In the following we shall assume, in the Jordan frame, the Friedmann-Lemaitre-Robertson-Walker (FLRW) background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j . \quad (3.1)$$

Now, let us consider the following conformal transformation:

$$\begin{aligned} ds^2 \rightarrow d\hat{s}^2 &= \Omega^2 ds^2 = \Omega^2 \{g_{00}(dt)^2 + 2g_{0i} dt dx^i + g_{ij} dx^i dx^j\} \\ &= g_{00}(\hat{dt})^2 + 2\Omega g_{0i} \hat{dt} dx^i + \Omega^2 g_{ij} dx^i dx^j \end{aligned} \quad (3.2)$$

where i and j are spatial indices and we have changed the time variable via the formula

$$\hat{dt} = \Omega dt = \Omega(\varphi(t)) dt . \quad (3.3)$$

The functional form of Ω will be determined below, but note that $\Omega(\varphi(t))$ is a classical quantity, i.e., it does not contain a fluctuation part (see Eq. (4.4)). We also assume $\Omega(\varphi(t)) > 0$ for the solution $\varphi(t)$. This assumption could be related with the stability of the ground state. Although the stability has been investigated in the literature [22, 23] for a particular choice of $f(\varphi)$ and is interesting in its own right, we do not go into the details here.

We are thus led to redefine the metric in the following way:

$$\hat{g}_{00} = g_{00}, \quad \hat{g}_{0i} = \Omega g_{0i}, \quad \hat{g}_{ij} = \Omega^2 g_{ij}, \quad (i, j = 1, 2, 3) . \quad (3.4)$$

The FLRW background turns out in the Einstein frame to be

$$d\hat{s}^2 = \Omega^2 ds^2 = -\hat{dt}^2 + \hat{a}^2(\hat{t})\delta_{ij}dx^i dx^j . \quad (3.5)$$

where our notation is

$$\hat{a}(\hat{t}) = \Omega(\varphi(t)) a(t) , \quad (3.6)$$

and we should note again that $\hat{a}(\hat{t})$ does not contain a fluctuation part. Starting from the action (2.1) and using the conformal transformation properties of the Ricci scalar [30], [31]

$$R = \Omega^2 \left[\hat{R} + 6\hat{\square} \ln \Omega - 6\hat{g}^{\mu\nu} (\partial_\mu \ln \Omega) (\partial_\nu \ln \Omega) \right] , \quad (3.7)$$

we can show that the action (2.1) is transformed as follows:

$$\begin{aligned} S &= \frac{1}{2} \int \hat{dt} d^3\vec{x} \Omega^{-4} \sqrt{-\hat{g}} \\ &\times \left[\Omega^2 f(\varphi) \left\{ \hat{R} + 6\hat{\square} \ln \Omega - 6\hat{g}^{\mu\nu} (\partial_\mu \ln \Omega) (\partial_\nu \ln \Omega) \right\} + 2P(\varphi, X) \right] . \end{aligned} \quad (3.8)$$

In order to arrive at the Einstein frame by making the coefficient of the scalar curvature unity, we set

$$\Omega(\varphi) = \sqrt{f(\varphi)} . \quad (3.9)$$

This puts the action into the following form:

$$S = \frac{1}{2} \int dt d^3\vec{x} \sqrt{-\hat{g}} \left[\hat{R} + 2\hat{P}(\varphi, \hat{X}) \right] \quad (3.10)$$

where in the last step above the $\hat{\square}$ term disappears after integrating by parts and we have also defined

$$\hat{X} = -\frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi = \frac{1}{\Omega^2} X, \quad (3.11)$$

$$\begin{aligned} \hat{P}(\varphi, \hat{X}) &= \frac{1}{\Omega^4} P(\varphi, X) - 3\hat{g}^{\mu\nu} (\partial_\mu \ln \Omega) (\partial_\nu \ln \Omega) \\ &= \frac{1}{\Omega^4} P(\varphi, X) + \frac{6\hat{X}}{\Omega^2} \left(\frac{d\Omega}{d\varphi} \right)^2. \end{aligned} \quad (3.12)$$

The energy-momentum tensor of φ is given in terms of $\hat{P}(\varphi, \hat{X})$ as

$$T^0_0 \equiv \hat{E} = 2\hat{X} \hat{P}_{,\hat{X}} - \hat{P}, \quad T^i_j = -\hat{P} \delta^i_j, \quad (i, j = 1, 2, 3).$$

In our abbreviated notation, $\hat{P}_{,\hat{X}}$ is the derivative of \hat{P} with respect to \hat{X} . The energy conservation law and the Friedmann equation are given as usual by

$$\frac{d\hat{E}}{dt} = -3\hat{H} (\hat{E} + \hat{P}), \quad \hat{H}^2 \equiv \left(\frac{1}{\hat{a}} \frac{d\hat{a}}{dt} \right)^2 = \frac{\hat{E}}{3}. \quad (3.13)$$

4. ADM formalism in the Einstein frame

Cosmological perturbation theories have been developed for many years and the reader is referred to [32] for reviews. In the case of the general quantum computation in gravity, we first decompose the metric perturbations using the ADM formalism [33], an application of which to cosmology has been done very early by Bardeen [34]. This method of decomposition has recently been applied to the study of non-Gaussianity by Maldacena [7] in standard inflation and was then considered for more general models in [19, 20, 35]. Because we shall work in the Einstein frame we can simply use the results already derived in [19, 20, 35] for example, but with quantities now expressed with hats ($\hat{}$). Thus, we shall just briefly outline the method, which provides us with a second order action for the curvature perturbation (first derived by different means in [29]) and together with a third order action which leads to the bispectrum after applying the in-in formalism along lines similar to [7].

We consider the metric which may be given as

$$d\hat{s}^2 = -\hat{N}^2 (d\hat{t})^2 + \hat{h}_{ij} \left(dx^i + \hat{N}^i d\hat{t} \right) \left(dx^j + \hat{N}^j d\hat{t} \right). \quad (4.1)$$

The action in terms of the ADM variables is expressed as

$$S = \frac{1}{2} \int d\hat{t} d^3\vec{x} \sqrt{\hat{h}} \hat{N} \left\{ {}^{(3)}\hat{R} + \frac{1}{\hat{N}^2} \left(\hat{E}_{ij} \hat{E}^{ij} - \hat{E}^2 \right) + 2\hat{P}(\varphi, \hat{X}) \right\}, \quad (4.2)$$

where $\hat{h} = \det \hat{h}_{ij}$, and the tensor \hat{E}_{ij} and \hat{E} are defined, respectively, by

$$\hat{E}_{ij} = \frac{1}{2} \left(\frac{\partial \hat{h}_{ij}}{\partial \hat{t}} - \hat{\nabla}_i \hat{N}_j - \hat{\nabla}_j \hat{N}_i \right), \quad \hat{E} = \hat{E}_{ij} \hat{h}^{ij}. \quad (4.3)$$

The covariant derivative $\hat{\nabla}_i$ is with respect to \hat{h}_{ij} . The three dimensional scalar curvature is denoted as ${}^{(3)}\hat{R}$

We choose a gauge to fix time and spatial reparametrization by setting [7]

$$\delta\varphi = 0. \quad (4.4)$$

together with fluctuations around the FLRW metric in the following form:

$$d\hat{s}^2 = -(d\hat{t})^2 + \hat{a}(\hat{t})^2 e^{2\hat{\mathcal{R}}} (\delta_{ij} + \hat{\gamma}_{ij}), \quad (\partial^i \hat{\gamma}_{ij} = 0, \quad \delta^{ij} \hat{\gamma}_{ij} = 0). \quad (4.5)$$

Eqs. (4.4) and (4.5) fix the whole gauge degrees of freedom [7, 36]. Eq. (4.5) should be compared with the fluctuations in the Jordan frame

$$\begin{aligned} ds^2 &= -(dt)^2 + a(t)^2 e^{2\mathcal{R}} (\delta_{ij} + \gamma_{ij}) \quad (\partial^i \gamma_{ij} = 0, \quad \delta^{ij} \gamma_{ij} = 0), \\ &= \frac{1}{\Omega^2} \{ -(dt)^2 + \hat{a}(\hat{t})^2 e^{2\hat{\mathcal{R}}} (\delta_{ij} + \gamma_{ij}) \}, \end{aligned} \quad (4.6)$$

where we have substituted the relations (3.3) and (3.6). Eqs. (4.5) and (4.6) indicate clearly that we need not distinguish \mathcal{R} from $\hat{\mathcal{R}}$ or γ_{ij} from $\hat{\gamma}_{ij}$ (as stressed in [12, 13]), i.e.,

$$\hat{\mathcal{R}} = \mathcal{R}, \quad \hat{\gamma}_{ij} = \gamma_{ij}. \quad (4.7)$$

Hereafter we consider only the curvature perturbation $\hat{\mathcal{R}} = \mathcal{R}$, and leave discussion of the tensor perturbation $\hat{\gamma}_{ij} = \gamma_{ij}$ for future work.

The equations of motion for \hat{N} and \hat{N}^i are the Hamiltonian and momentum constraints which are given by

$$\hat{\nabla}_i \left\{ \frac{1}{\hat{N}} \left(\hat{E}_j^i - \delta_j^i \hat{E} \right) \right\} = 0, \quad (4.8)$$

$${}^{(3)}\hat{R} + 2\hat{P} - 4\hat{X}\hat{P}_{,\hat{X}} - \frac{1}{\hat{N}^2} \left(\hat{E}_{ij}\hat{E}^{ij} - \hat{E}^2 \right) = 0. \quad (4.9)$$

We can solve the Hamiltonian and momentum constraints order by order, by setting

$$\hat{N} = 1 + \hat{N}^{(1)} + \dots, \quad \hat{N}_i = \partial_i \hat{\psi}^{(1)} + \hat{\tilde{N}}_i^{(1)} + \dots, \quad (4.10)$$

where $\hat{N}^{(1)}$, $\hat{\psi}^{(1)}$ and $\hat{\tilde{N}}_i^{(1)}$ are the first order and the ellipsis denotes higher order terms. Here we assume $\partial^i \hat{\tilde{N}}_i^{(1)} = 0$. The solutions to (4.8) and (4.9) have been worked out [19, 20] and are known to be

$$\hat{N}^{(1)} = \frac{1}{\hat{H}} \frac{\partial \hat{\mathcal{R}}}{\partial \hat{t}}, \quad \hat{\tilde{N}}_i^{(1)} = 0, \quad \hat{\psi}^{(1)} = -\frac{\hat{\mathcal{R}}}{\hat{H}} + \hat{\chi}, \quad \left(\hat{H} = \frac{1}{\hat{a}} \frac{d\hat{a}}{d\hat{t}} \right). \quad (4.11)$$

Here $\hat{\chi}$ is a solution to the differential equation

$$\partial^2 \hat{\chi} = \hat{a}^2 \frac{\hat{\varepsilon}}{\hat{c}_s^2} \frac{d\hat{\mathcal{R}}}{dt}, \quad (\partial^2 = \delta^{ij} \partial_i \partial_j), \quad (4.12)$$

where the slow variation parameter and the sound speed are defined, respectively, by

$$\hat{\varepsilon} = -\frac{1}{\hat{H}^2} \frac{d\hat{H}}{dt}, \quad \hat{c}_s^2 = \frac{\hat{P}_{,\hat{X}}}{2\hat{X}\hat{P}_{,\hat{X}\hat{X}} + \hat{P}_{,\hat{X}}}. \quad (4.13)$$

As argued in [7, 20], when calculating the effective action up to third order in $\hat{\mathcal{R}}$, we need not calculate the $\hat{\mathcal{R}}^2$ terms in \hat{N} or \hat{N}_i .

Now let us turn to expansion of the action (2.1) in power series of the curvature perturbation

$$S = S^{(0)} + S^{(2)} + S^{(3)} + \dots, \quad (4.14)$$

where $S^{(n)}$ is the n -th order action. Seery and Lidsey [19] and Chen et al. [20] have studied $S^{(2)}$ and $S^{(3)}$ after performing integration by parts. After straightforward calculations they obtained

$$\begin{aligned} S^{(2)} &= \int dt d^3\vec{x} \left\{ \hat{a}^3 \frac{\hat{\varepsilon}}{\hat{c}_s^2} \left(\frac{d\hat{\mathcal{R}}}{dt} \right)^2 - \hat{a}\hat{\varepsilon} (\partial\hat{\mathcal{R}})^2 \right\} \\ &= \int dt d^3\vec{x} \left\{ \hat{a}^3 \frac{\hat{\Sigma}}{\hat{H}^2} \left(\frac{d\hat{\mathcal{R}}}{dt} \right)^2 - \hat{a}\hat{\varepsilon} (\partial\hat{\mathcal{R}})^2 \right\}, \end{aligned} \quad (4.15)$$

and

$$\begin{aligned} S^{(3)} &= \int dt d^3\vec{x} \left[-\hat{a}\hat{\varepsilon}\hat{\mathcal{R}}(\partial\hat{\mathcal{R}})^2 - (\hat{\Sigma} + 2\hat{\lambda}) \frac{\hat{a}^3}{\hat{H}^3} \left(\frac{d\hat{\mathcal{R}}}{dt} \right)^3 + \frac{3\hat{a}^3\hat{\varepsilon}}{\hat{c}_s^2} \hat{\mathcal{R}} \left(\frac{d\hat{\mathcal{R}}}{dt} \right)^2 \right. \\ &\quad + \frac{1}{2\hat{a}} \left(3\hat{\mathcal{R}} - \frac{1}{\hat{H}} \frac{d\hat{\mathcal{R}}}{dt} \right) (\partial_i \partial_j \hat{\psi}^{(1)} \partial^i \partial^j \hat{\psi}^{(1)} - \partial^2 \hat{\psi}^{(1)} \partial^2 \hat{\psi}^{(1)}) \\ &\quad \left. - \frac{2}{\hat{a}} (\partial_i \hat{\psi}^{(1)}) (\partial^i \hat{\mathcal{R}}) (\partial^2 \hat{\psi}^{(1)}) \right], \end{aligned} \quad (4.16)$$

which we have also confirmed. Here we use notation analogous to that in [19] and [20], i.e.,

$$\hat{\Sigma} = \hat{X} \hat{P}_{,\hat{X}} + 2\hat{X}^2 \hat{P}_{,\hat{X}\hat{X}} = \frac{\hat{X} \hat{P}_{,\hat{X}}}{\hat{c}_s^2} = \frac{\hat{\varepsilon} \hat{H}^2}{\hat{c}_s^2}, \quad (4.17)$$

$$\hat{\lambda} = \hat{X}^2 \hat{P}_{,\hat{X}\hat{X}} + \frac{2}{3} \hat{X}^3 \hat{P}_{,\hat{X}\hat{X}\hat{X}}. \quad (4.18)$$

Note that \hat{X} , $\hat{P}_{,\hat{X}}$ and $\hat{P}_{,\hat{X}\hat{X}}$ are all understood as classical values corresponding to $\varphi = \varphi(t)$. For possible operators that could appear in the third order action, see Ref. [37].

5. The Jordan frame

5.1 The quadratic part of the action

The definition (4.17) of $\widehat{\Sigma}$ together with (3.12) enables us to express $\widehat{\Sigma}$ in terms of quantities without hat:

$$\begin{aligned}\widehat{\Sigma} &= \frac{1}{\Omega^4} \left\{ X P_{,X} + 2X^2 P_{,XX} + 6X \left(\frac{d\Omega}{d\varphi} \right)^2 \right\} \\ &= \frac{1}{f(\varphi)^2} \left\{ \Sigma + \frac{3}{4f(\varphi)} \left(\frac{df}{dt} \right)^2 \right\} \\ &= e^{-4\theta} a^4 \left\{ \Sigma + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} .\end{aligned}\tag{5.1}$$

Here we have introduced the notation

$$\Sigma = X P_{,X} + 2X^2 P_{,XX} , \quad \theta = \frac{1}{2} \log(f(\varphi) a^2), \quad H = \frac{1}{a} \frac{da}{dt} .\tag{5.2}$$

As we see Σ is an analog in the Jordan frame, corresponding to $\widehat{\Sigma}$. The function θ has been employed by Qiu and Yang [21] extensively. The Hubble constant defined in the Jordan frame is denoted by H .

The connection between \widehat{H} and the Hubble constant H in the Jordan frame is easily seen from (3.6) to be

$$\widehat{H} = \frac{1}{\widehat{a}} \frac{d\widehat{a}}{dt} = \frac{H}{\Omega} + \frac{1}{\Omega^2} \frac{d\Omega}{dt} = \frac{1}{\sqrt{f(\varphi)}} \left\{ H + \frac{1}{2f(\varphi)} \frac{df(\varphi)}{dt} \right\} = e^{-\theta} a \frac{d\theta}{dt} .\tag{5.3}$$

One of the standard parameters $\widehat{\varepsilon}$ in the Einstein frame can be expressed in terms of θ by using (5.3) as

$$\widehat{\varepsilon} = -\frac{1}{\widehat{H}^2} \frac{d\widehat{H}}{dt} = -\left(\frac{d\theta}{dt} \right)^{-2} \left\{ \frac{d^2\theta}{dt^2} + H \frac{d\theta}{dt} - \left(\frac{d\theta}{dt} \right)^2 \right\} .\tag{5.4}$$

The coefficients of each term in the quadratic action (4.15) can all be expressed in terms of θ ,

$$\widehat{a}^3 \frac{\widehat{\Sigma}}{\widehat{H}^2} = e^\theta a^2 \left\{ \Sigma + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-2} ,\tag{5.5}$$

$$-\widehat{a}\widehat{\varepsilon} = e^\theta \left(\frac{d\theta}{dt} \right)^{-2} \left\{ \frac{d^2\theta}{dt^2} + H \frac{d\theta}{dt} - \left(\frac{d\theta}{dt} \right)^2 \right\} .\tag{5.6}$$

We thus end up with the quadratic action

$$\begin{aligned}S^{(2)} &= \int dt d^3\vec{x} \left[a^3 \left\{ \Sigma + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-2} \left(\frac{d\mathcal{R}}{dt} \right)^2 \right. \\ &\quad \left. + e^{2\theta} a^{-1} \left(\frac{d\theta}{dt} \right)^{-2} \left\{ \frac{d^2\theta}{dt^2} + H \frac{d\theta}{dt} - \left(\frac{d\theta}{dt} \right)^2 \right\} (\partial\mathcal{R})^2 \right]\end{aligned}\tag{5.7}$$

in the Jordan frame. This agrees with the result obtained previously by Qiu and Yang [21] via a direct computation.

5.2 The cubic part of the action

We would also like to express the cubic action (4.16) by using quantities without hat. The definition (4.18) is shuffled into

$$\hat{\lambda} = \frac{1}{\Omega^4} \left(X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX} \right) \equiv e^{-4\theta} a^4 \lambda, \quad (5.8)$$

where λ , an analog in the Jordan frame, is defined by the equation above. The coefficient of the second term in the cubic action (4.16) thus turns out to be

$$\left(\hat{\Sigma} + 2\hat{\lambda} \right) \frac{\hat{a}^3}{\hat{H}^3} = a e^{2\theta} \left\{ \Sigma + 2\lambda + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-3}. \quad (5.9)$$

The quantities that appear in the third and fourth terms in (4.16) are also worked out straightforwardly. In fact a combined use of (4.17) and (5.5) gives us

$$\begin{aligned} \frac{3\hat{a}^3 \varepsilon}{\hat{c}_s^2} &= \frac{3\hat{a}^3 \hat{\Sigma}}{\hat{H}^2} \\ &= 3e^\theta a^2 \left\{ \Sigma + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-2}. \end{aligned} \quad (5.10)$$

From (4.7) and (5.3), we also get

$$3\hat{\mathcal{R}} - \frac{1}{\hat{H}} \frac{d\hat{\mathcal{R}}}{dt} = 3\mathcal{R} - \left(\frac{d\theta}{dt} \right)^{-1} \frac{d\mathcal{R}}{dt}. \quad (5.11)$$

We are thus led to the cubic action

$$\begin{aligned} S^{(3)} &= \int dt d^3 \vec{x} \left[e^{2\theta} a^{-1} \left(\frac{d\theta}{dt} \right)^{-2} \left\{ \frac{d^2 \theta}{dt^2} + H \frac{d\theta}{dt} - \left(\frac{d\theta}{dt} \right)^2 \right\} \mathcal{R} (\partial \mathcal{R})^2 \right. \\ &\quad - a^3 \left\{ \Sigma + 2\lambda + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-3} \left(\frac{d\mathcal{R}}{dt} \right)^3 \\ &\quad + 3a^3 \left\{ \Sigma + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-2} \mathcal{R} \left(\frac{d\mathcal{R}}{dt} \right)^2 \\ &\quad + \frac{1}{2} e^{2\theta} a^{-3} \left\{ 3\mathcal{R} - \left(\frac{d\theta}{dt} \right)^{-1} \frac{d\mathcal{R}}{dt} \right\} \left(\partial_i \partial_j \psi^{(1)} \partial^i \partial^j \psi^{(1)} - \partial^2 \psi^{(1)} \partial^2 \psi^{(1)} \right) \\ &\quad \left. - 2e^{2\theta} a^{-3} (\partial_i \psi^{(1)}) (\partial^i \mathcal{R}) (\partial^2 \psi^{(1)}) \right], \end{aligned} \quad (5.12)$$

which is suitable for computation of cosmological correlations in the Jordan frame. In the above we have set

$$\hat{\psi}^{(1)} = \Omega \psi^{(1)} = e^\theta a^{-1} \psi^{(1)}. \quad (5.13)$$

Note that Eq. (5.13) is due to the fact that the lapse function, shift function and the three-dimensional metric in Einstein and Jordan frames are connected by the formulae

$$\hat{N} = N, \quad \hat{N}_i = \Omega N_i, \quad \hat{N}^i = \frac{1}{\Omega} N^i, \quad \hat{h}_{ij} = \Omega^2 h_{ij}, \quad (5.14)$$

in accordance with (3.4). The cubic action (5.12) agrees with the result in [21] obtained by a direct method.

6. Frame independence of quantum calculations

As a final step of our analyses let us scrutinize the properties of quantization in the Einstein and Jordan frames. The Lagrangian densities in Einstein and Jordan frames are connected by the relation $\Omega \hat{\mathcal{L}} = \mathcal{L}$. This can be seen by rewriting the action in the following way:

$$S = \int d\hat{t} d^3 \vec{x} \hat{\mathcal{L}} = \int dt d^3 \vec{x} \Omega \hat{\mathcal{L}} = \int dt d^3 \vec{x} \mathcal{L}. \quad (6.1)$$

The canonical momentum variables conjugate to $\hat{\mathcal{R}}$ and \mathcal{R} , i.e.,

$$\hat{\Pi} = \frac{\delta \hat{\mathcal{L}}}{\delta(\partial \hat{\mathcal{R}} / \partial \hat{t})}, \quad \Pi = \frac{\delta \mathcal{L}}{\delta(\partial \mathcal{R} / \partial t)} \quad (6.2)$$

are also apparently identical with each other, namely,

$$\hat{\Pi}(\hat{t}, \vec{x}) = \Pi(t, \vec{x}). \quad (6.3)$$

The canonical commutation relations

$$[\hat{\mathcal{R}}(\hat{t}, \vec{x}), \hat{\Pi}(\hat{t}, \vec{x}')] = i \delta^3(\vec{x} - \vec{x}'), \quad [\mathcal{R}(t, \vec{x}), \Pi(t, \vec{x}')] = i \delta^3(\vec{x} - \vec{x}') \quad (6.4)$$

lead to the same quantization procedure common in both Einstein and Jordan frames.

Let us turn to the mode expansion in Einstein and Jordan frames,

$$\hat{\mathcal{R}}(\hat{t}, \vec{x}) = \int \frac{d^3 \vec{k}}{(2\pi)^3} \left\{ \hat{a}(\vec{k}) \hat{u}(\hat{t}, \vec{k}) e^{i\vec{k} \cdot \vec{x}} + \hat{a}^\dagger(\vec{k}) \hat{u}^*(\hat{t}, \vec{k}) e^{-i\vec{k} \cdot \vec{x}} \right\}, \quad (6.5)$$

$$\mathcal{R}(t, \vec{x}) = \int \frac{d^3 \vec{k}}{(2\pi)^3} \left\{ a(\vec{k}) u(t, \vec{k}) e^{i\vec{k} \cdot \vec{x}} + a^\dagger(\vec{k}) u^*(t, \vec{k}) e^{-i\vec{k} \cdot \vec{x}} \right\}, \quad (6.6)$$

and look at the connection between creation and annihilation operators in the two frames. Hereafter all quantities are supposed to be in the interaction picture. The differential equations to be satisfied by the mode functions are given by the quadratic part of the action and are

$$\left\{ \frac{d}{d\hat{t}} \left(\hat{a}^3 \frac{\hat{\Sigma}}{\hat{H}^2} \frac{d}{d\hat{t}} \right) + \hat{a} \hat{\varepsilon} \vec{k}^2 \right\} \hat{u}(\hat{t}, \vec{k}) = 0 \quad (6.7)$$

and

$$\begin{aligned} \frac{d}{dt} \left[a^3 \left\{ \Sigma + 3e^{2\theta} a^{-2} \left(\frac{d\theta}{dt} - H \right)^2 \right\} \left(\frac{d\theta}{dt} \right)^{-2} \frac{d}{dt} \right] u(t, \vec{k}) \\ - e^{2\theta} a^{-1} \left(\frac{d\theta}{dt} \right)^{-2} \left\{ \frac{d^2\theta}{dt^2} + H \frac{d\theta}{dt} - \left(\frac{d\theta}{dt} \right)^2 \right\} \vec{k}^2 u(t, \vec{k}) = 0 \end{aligned} \quad (6.8)$$

in the two frames, respectively. Qiu and Yang [21] elaborated the solutions to (6.8) in the Jordan frame. It should be, however, remarked that Eq. (6.8) can be equivalently rewritten as

$$\left\{ \frac{1}{\Omega} \frac{d}{dt} \left(\hat{a}^3 \frac{\hat{\Sigma}}{\hat{H}^3} \frac{1}{\Omega} \frac{d}{dt} \right) + \hat{a} \hat{\varepsilon} \vec{k}^2 \right\} u(t, \vec{k}) = 0 , \quad (6.9)$$

thanks to the relation (5.5) and (5.6). Furthermore, it is also obvious that (6.9) is the same differential equation as (6.7) because of (3.3).

The solutions to (6.8) can be worked out by going over to the Einstein frame, thereby making full use of results obtained previously. In fact, by adjusting the normalization of $u(t, \vec{k})$ and $\hat{u}(\hat{t}, \vec{k})$ in the same manner, we can always set

$$\hat{u}(\hat{t}, \vec{x}) = u(t, \vec{x}) , \quad (6.10)$$

$$\hat{a}(\vec{k}) = a(\vec{k}), \quad \hat{a}^\dagger(\vec{k}) = a^\dagger(\vec{k}) . \quad (6.11)$$

The “free vacuum” $|0\rangle$ is defined as the state that is annihilated by all of the annihilation operators, i.e., $a(\vec{k})|0\rangle = 0$, and $\hat{a}(\vec{k})|0\rangle = 0$. Because of (6.11), the free vacuum $|0\rangle$ is common in both Einstein and Jordan frames.

In the interaction picture, the time-development of the “interaction vacuum” starting from $|0\rangle$ at some early time (\hat{t}_0 or t_0) is described by the unitary transformation composed of the interaction Hamiltonian, that is,

$$|\widehat{\text{vac}}\rangle = T \exp \left(-i \int_{\hat{t}_0}^{\hat{t}} d\hat{t}' \hat{H}_{\text{int}}(\hat{t}') \right) |0\rangle \quad (6.12)$$

in the Einstein frame and

$$|\text{vac}\rangle = T \exp \left(-i \int_{t_0}^t dt' H_{\text{int}}(t') \right) |0\rangle \quad (6.13)$$

in the Jordan frame. Here T indicates the time-ordering and the interaction Hamiltonian is denoted by $\hat{H}_{\text{int}}(\hat{t})$ ($H_{\text{int}}(t)$) in the Einstein (Jordan) frame. Now we show that the two states, (6.12) and (6.13), are in fact identical. From the preceding analyses, we can easily see that $\hat{H}_{\text{int}}(\hat{t})$ is connected with $H_{\text{int}}(t)$ via

$$\hat{H}_{\text{int}}(\hat{t}) = \frac{1}{\Omega(\varphi(t))} H_{\text{int}}(t) . \quad (6.14)$$

Combining (6.14) with (3.3), we obtain a simple relation:

$$\int_{\hat{t}_0}^{\hat{t}} d\hat{t}' \hat{H}_{\text{int}}(\hat{t}') = \int_{t_0}^t dt' H_{\text{int}}(t') , \quad (6.15)$$

and therefore the two vacua are identical, i.e., $|\widehat{\text{vac}}\rangle = |\text{vac}\rangle$.

In Sec. 5 we discussed only the quadratic and cubic terms, $S^{(2)}$ and $S^{(3)}$ in the action, but our argument can obviously be extended to higher order terms in $\widehat{\mathcal{R}} = \mathcal{R}$. Since the interaction vacuum is one and the same in both frames, the cosmological correlation functions computed in each frame can be shown to agree with each other, that is,

$$\langle \widehat{\text{vac}} | \widehat{\mathcal{R}}(\widehat{t}, \vec{k}_1) \cdots \widehat{\mathcal{R}}(\widehat{t}, \vec{k}_N) | \widehat{\text{vac}} \rangle = \langle \text{vac} | \mathcal{R}(t, \vec{k}_1) \cdots \mathcal{R}(t, \vec{k}_N) | \text{vac} \rangle \quad (6.16)$$

for an arbitrary integer N , not to mention the $N = 2$ (power spectrum) and $N = 3$ (bispectrum) cases. Here $\mathcal{R}(t, \vec{k})$ ($\widehat{\mathcal{R}}(\widehat{t}, \vec{k})$) is the three-dimensional Fourier transform of \mathcal{R} ($\widehat{\mathcal{R}}$) in the Jordan (Einstein) frame.

7. Summary

In the present paper we have made an overall comparison between two types of cosmological perturbation theories, one in the Einstein and the other in the Jordan frame. By virtue of the basic relation (4.7), the connection between the two frames is extremely simple and we can easily go back and forth between the two frames. In this way we are able to find a way for alleviating the need for separate studies in the two frames.

In the actual calculation of the correlation (6.16), we have to evaluate multiple-commutators [38]

$$\begin{aligned} & \langle \widehat{\text{vac}} | \widehat{\mathcal{R}}(\widehat{t}, \vec{k}_1) \cdots \widehat{\mathcal{R}}(\widehat{t}, \vec{k}_N) | \widehat{\text{vac}} \rangle \\ &= \sum_{n=0}^{\infty} i^n \int_{\widehat{t}_0}^{\widehat{t}} d\widehat{t}_n \int_{\widehat{t}_0}^{\widehat{t}_n} d\widehat{t}_{n-1} \cdots \int_{\widehat{t}_0}^{\widehat{t}_2} d\widehat{t}_1 \\ & \quad \times \langle 0 | [\widehat{H}_{\text{int}}(\widehat{t}_n), [\widehat{H}_{\text{int}}(\widehat{t}_{n-1}), \cdots [\widehat{H}_{\text{int}}(\widehat{t}_1), \widehat{\mathcal{R}}(\widehat{t}, \vec{k}_1) \cdots \widehat{\mathcal{R}}(\widehat{t}, \vec{k}_N)] \cdots]] | 0 \rangle \end{aligned} \quad (7.1)$$

in the Einstein frame or similarly in the Jordan frame. Thanks to (6.15) every step of calculating the commutators in the Einstein and Jordan frames may be compared with each other and can be shown to be identical.

Acknowledgements

The authors would like to express their sincere thanks to Professor Tetsuya Onogi for collaboration in the early stages of the present work. They are also grateful to Professor Misao Sasaki and Dr. Masato Minamitsuji for useful discussions. The authors' cordial thanks should also go to Professor Takeshi Chiba and Professor Masahide Yamaguchi for calling their attention to Ref. [12].

Note added : After sending the present paper to the arXiv, we were informed by Dr. Yun-Song Piao of his recent work [39]. He has shown that the primordial curvature perturbation and tensor perturbation generated during two conformally related backgrounds are equal. We were also informed by Dr. Sandipan Kundu of his recent work [40], in which

the behavior of the curvature perturbation under the three-dimensional rescaling and its implications to non-Gaussianity are discussed. We would like to thank Dr. Yun-Song Piao and Dr. Sandipan Kundu for their kind communication.

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